CFDSC Bulletin

Société canadienne de CFD Society of Canada

CFD Society of Canada | Société Canadienne de CFD

Nov. 2006 | Issue No. 18

In this Issue

1

President's Message

The short hot summer of 2006 will be well remembered by the CFD Society of Canada as the time when we participated in a well organized, carefully structured and successful annual conference...

2

CFD Society of Canada

The Computational Fluid Dynamic Society of Canada (CFDSC) promotes the application of computational fluid dynamics (CFD) to provide a better understanding of fluid dynamic…

3

CFDSC Scholarship Applications

Budget permitting, the CFD Society of Canada will award one or more scholarships per year to successful applicant(s) conducting research work related to computational fluid dynamics.

4

Call for Lifetime Achievement Award Nominations

CFD Society of Canada is now accepting nominations for the presentation of our 2007 annual Lifetime Achievement Award, awarded to scientists recognized

5

Call for Papers | CFD2007

The CFD Society of Canada is pleased to announce that its 15th Annual Conference will be held in Toronto, Ontario, Canada, from Sunday May 27, 2007 until...

6

Lifetime Achievement Award

Wagdi Habashi was born in Egypt and came to Canada at the age of 18, in 1964. Fred, as they called him since birth even in Egypt, had entered Cairo University at the age of 16 and completed...

7

A report on CFD 2006

The 14th Annual Conference of the CFDSC was held on the campus of Queen's University in July. By all accounts, it was a...

8

CFDSC Graduate Scholarship

In 2005, the CFD Society of Canada created a new scholarship for full time graduate students at Canadian universities that are conducting...

9

Two Dimensional Flow over Rough Topology

Spectral Analysis

The primary goal of this work is to provide a description of the energy cascade in internal waves generated by tidal flow...

PRESIDENT'S MESSAGE

by

Mahmood Khalid

The short hot summer of 2006 will be well remembered by the CFD Society of Canada as the time when we participated in an organized, carefully structured and successful annual conference on the grounds of the Queens University in Kingston. Some 130 delegates were welcomed from at least 15 countries. The conference covered a wide range of topics including, Rotating Machinery, Biofluids, Fuel Cells, Combustion, Data Mining, Visualization techniques and other CFD related topics. The highlight of the social events was a boat tour of the 1000 Islands which indeed was also the venue for the main event of the evening in which Prof. Fred Habashi was honored with a plaque for his recognition as the recipient of the Society's Lifetime Achievement Award for the year 2006. Our Kudos to Prof. Pollard and the crew for catering to pick up those members who actually missed the boat. The entertainments also included a skit by a local performer who along with a lady companion acted out various anecdotes from the life history of a well known Canadian Prime Minster, Sir John A McDonald. It was very much an informative exercise for me to learn about the energy and dedication of an outstanding prime minister. A rare breed I am sure. I was given an unprecedented mandate to be the President of your society for the third year and I hope very much to be able to complete a number of items being pursued by the Board. Our relationship with IJCFD has now been well established with almost regular appearance of a cross section of our papers as a special issue and our Bulletin is becoming a regular respected bi-annual event with at least one or two journal quality articles. Building on this successful conference we would like very much for you to keep your calendar dates open for our next year's conference to be held in Toronto during the week of 27th May 2007.

CFD Society of Canada

Objectives ● The Computational Fluid Dynamic Society of Canada (CFDSC) promotes the application of computational fluid dynamics (CFD) to provide a better understanding of fluid dynamic processes in all applicable areas and thereby supports the competitiveness of Canadian industry and advances knowledge in the field. These objectives are met by *(1)* establishing a network of CFD practitioners and developers from industry, government and universities, *(2)* identifying national needs and priorities for research in CFD, *(3)* promoting research in CFD and related areas, *(4)* enhancing industry capabilities in CFD by the education and training of professionals, *(5)* promoting the use and increasing the availability of computational tools for CFD, *(6)* promoting communication and exchange within CFD and related disciplines (e.g. environment, biology and chemistry), *(7)* organizing conferences and seminars, and *(8)* representing Canada at international forums.

Membership Annual membership fees of the CFDSC are $50 for regular members, $20 for student members, and $25 for members who are retired.

World Wide Web | www.cfdsc.ca

Address

CFD Society of Canada
P. O. Box 4871, Station E
Ottawa, Ontario, Canada KIS 5J1

Bulletin The Bulletin of the CFDSC is published about three times a year to provide members with information on current CFD issues. The published articles express the views and opinions of their respective authors and do not necessarily represent those of the CFDSC. The Managing Editors are:

Mahmood Khalid | NRC-IAR
Mobina Haider | Ottawa, Canada

mobina@inbox.com

Société Canadienne de CFD

Objectifs ● La Société canadienne de CFD (SCCFD) fait la promotion des applications de <<computational fluid dynamics>> afin de fournir une meilleure compréhension de la dynamique des fluides et contribue ainsi au développement de la compétitivité industrielle canadienne et à l'avancement des connaissances dans le domaine. Ces objectifs sont atteints en *1)* établissant un réseau d'utilisateurs et de développeurs de CFD dans les industries, les gouvernements et les universités, *2)* identifiant les besoins nationaux et les priorités de recherches en CFD, *3)* faisant la promotion de la recherche en CFD et les domaines reliés, *4)* rehaussant les capacités en CFD des industries par la formation et l'éducation des professionnels, *5)* faisant la promotion de l'utilisation et en augmentant la disponibilité d'outils informatiques en CFD, *6)* faisant la promotion de la communication et des échanges en CFD avec les disciplines reliées (par example, en environnement, en biologie et en chimie), *7)* organisant des conférences et des séminaires, *8)* représentant le Canada dans les forums internationaux.

Cotisation des membres Les frais annuels d'adhésion sont de 50$ pour les membres réguliers, 20$ pour les membres étudiants, et 25$ pour les membres à la retraite.

World Wide Web | www.cfdsc.ca

Adresse

Société canadienne de CFD
B. P. 4871, Station 'E'
Ottawa, Ontario,Canada K1S 5J1

Bulletin Le bulletin de la SCCFD est publié environ trois fois par année afin de fournir aux membres de l'information sur les grands courants reliés à la CFD. Les articles publiés représentent les opinions des auteurs respectifs et ne représentent pas nécessairement celles de la SCCFD. Les rédacteurs exécutifs sont:

Mahmood Khalid	CNRC-IAR
Mobina Haider	Ottawa, Canada

CFDSC Scholarship Applications

Budget permitting, the CFD Society of Canada will award one or more scholarships per year to successful applicant(s) conducting research work related to computational fluid dynamics. The field of study may involve the development of CFD techniques and/or the use of CFD techniques to solve a particular problem. The applicant must be a full time student in a Masters or Ph.D. level graduate studies program at an accredited Canadian university. Applications can be found at the CFDSC website (http://www.cfdsc.ca/) and must be received by May 1st, 2007.

Call for Lifetime Achievement Award Nominations

CFD Society of Canada is now accepting nominations for the presentation of our next Lifetime Achievement Award, awarded to scientists recognized for their work in Computational Fluid Dynamics.

For specific information regarding eligibility criteria and nomination procedures please visit our website at http://www.cfdsc.ca.

CFDSC
Executive Committee & Directors

President
Mahmood Khalid - (2004-2007, G, NRC)
Executive Officer
Marilyn Lightstone - (2005-2008, U, McMaster)
Treasurer
François Lesage - (2003-2006, G, DND)
Communications Officer
Jon Pharoah - (2003-2006, U, Queens)
Membership Officer
Suzhen Chen - (2005-2008, G, NRC)
Secretary and Student Director
Anh Truong
Directors
Pengfei Liu (2004-2007, G, NRC)
Anh Truong (student)
Eric Thornhill (2005-2008, G, DRDC)
Jacek Szymanski (2004-2007, I, Nuclear Safety Solutions)
Clinton Groth (2005-2008, U, Toronto)
Dave Whitehouse (2005-2008, I, Martec)

U : University | G : Government | I : Industry

Call for Papers | CFD2007
www.cfd2007.org
info@cfd2007.org

15th Annual Conference of the CFD Society of Canada (CFD2007)

The CFD Society of Canada is pleased to announce that its 15th Annual Conference will be held in Toronto, Ontario, Canada, from Sunday May 27, 2007 until Thursday May 31, 2007. The conference will be held in the heart of downtown Toronto at the University of Toronto

89 Chestnut Residences. The conference will span three full days and will feature three keynote speakers: Profs. Bernardo Cockburn, Herman Deconinck, and Kemo Hanjalic. A one-day short course on Multidisciplinary Design Optimization (MDO) will be given by Prof. Joaquim Martins immediately following the technical program on Thursday May 31, 2007. The conference will feature a student paper competition and banquet. The latter will be held on Tuesday, May 29, 2007. Scope: The goal of CFD Society of Canada is to promote computational fluid dynamics and to provide a framework for communication and collaboration between researchers, developers and practitioners, in industry, government and academia. The annual conferences draw together researchers in CFD to highlight recent advances and challenges encountered in a broad spectrum of application areas and to facilitate scientific and technical exchange. Paper submissions are encouraged in all topics of current interest in the field of CFD. Topics of interest include but are not limited to: compressible, incompressible, multiphase, and free surface flows; reacting flows and combustion; flows in porous media; environmental and biological flows; turbulence and turbulence modeling; aero-elasticity and aero-acoustics; CFD-dominated multi-disciplinary analysis and optimization; algorithm development; mesh generation and adaptation; error estimation and control; and parallel algorithms.

Key Date and Deadlines

- Abstract submission deadline: December 15, 2006
- Notification of abstract acceptance: January 31, 2007
- Final paper submission deadline: April 16, 2007
- Early registration deadline: May 1, 2007

Call for Papers and Abstract Submission

Papers will be selected based upon extended abstracts. Authors are requested to submit extended abstracts in pdf format by uploading the documents to the conference web site, www.cfd2007.org, on or before December 15, 2006. The extended abstract is to be a minimum of three pages long and a maximum of four pages long. All abstracts will be peer reviewed. An electronic version of the conference proceedings will be produced and exceptional accepted papers will be considered for inclusion in a special edition of the International Journal of Computational Fluid Dynamics (IJCFD). Please visit the conference web site for complete details and instructions related to abstract submission and to download templates for abstract preparation.

Conference Co-Chairs

The conference will be co-chaired by Profs. Marilyn Lightstone (McMaster University), Jeffrey Yokota (Ryerson University), and Clinton Groth (University of Toronto).

Inquiries can be directed to the conference co-chairs by email at info@cfd2007.org.

We heartily invite you to submit an abstract to CFD2007,

Marilyn Lightstone, Jeffrey Yokota, and Clinton Groth, Conference Co-Chairs

Lifetime Achievement Award

Prof. Fred Habashi was duly honored with the Life Time Achievement Award of the CFD Society of Canada in July 2006.

Wagdi Habashi was born in Egypt and came to Canada at the age of 18, in 1964. Fred, as they called him since birth even in Egypt, had entered Cairo University at the age of 16 and completed 2 years of mechanical engineering before arriving here. At McGill, where he got admitted somewhat reluctantly, Egyptian education standards being still unknown, he excelled and graduated Summa Cum Laude in 1967, as first on the Engineering Faculty and meriting the British Association Medal of Lord Rankine. He obtained a Masters at McGill and was almost finished with his PhD in combustion when he decided that it would be best to try something else and departed to Cornell where he obtained a PhD in Aerospace for work on the Finite Element Method, with which he has been closely associated since. One of the tales Fred likes to tell is being admitted for graduate studies at MIT, Caltech, Princeton, Berkeley, Stanford and Cornell, but having his application rejected by the University of Toronto! He still has the letters, showing how unbelievable sometimes the rivalry can be.

After two years at Cornell, he accepted an Assistant Professorship position at the Stevens Institute of Technology, the birthplace of the ASME, and commuted between New Jersey and Ithaca to complete his Thesis. It was in 1975 that due to parental pressures he called Concordia during a visit to Montreal to enquire if they had positions, not knowing that on the same day CU had announced a vacancy in Thermofluids in the now defunct Montreal Star. Fred then spent 25 years at Concordia where the University's dynamism and flexibility allowed him to create a unique connection with industry, most notably Pratt & Whitney Canada where he maintained an office from 1977 to 2001 and with whose staff he has published over 90 technical papers, spanning, 2D, quasi-3D, and fully 3D potential, then Euler, then Navier-Stokes multistage turbomachinery.

In 2000 he moved to McGill which offered him the needed infrastructure to enlarge his CFD Laboratory activities by applying for and then establishing a CFI-sponsored supercomputer center (CLUMEQ, now in its second funding phase and the possible recipient of $30M within the National Platform Fund), as well as being the ideal environment to put in place an NSERC-Bombardier Industrial Research Chair in Multidisciplinary CFD long in the making.

Professor Habashi is the author of some 240 scientific papers; he is the Editor-in-Chief of the International Journal of Computational Fluid Dynamics and co-Editor of other book series. He is a Fellow of the Canadian Academy of Engineering, a Fellow of the American Society of Mechanical Engineers (ASME), an Associate Fellow of the American Institute of Aeronautics and Astronautics (AIAA). Dr. Habashi is a co-founder of the Canadian Society of Computational Fluid Dynamics (CSCFD) and a co-founder of CERCA, and was its Director Industry from 1992-95. Professor Habashi is also Honorary Professor of TongJi University in Shanghai.

He has received numerous scientific and technology transfer awards, among them the E.W.R. Steacie Fellowship from the Natural Sciences and Engineering Research Council of Canada (NSERC), the Technology Achievement Award of Pratt & Whitney Canada, the University Research Award of Concordia University, the Cray Gigaflop Award for the fastest computer code in the world. He was selected by the Quebec Science Magazine as one of *the 10 discoveries of 199*8 for his OptiMesh work, by the Montreal Gazette as one of the 10 top scientists in Montreal in its series, *Montreal the Year 2000*, and by the Canadian Foundation for Innovation as one of *Canada's top 25 scientists* in 2002. In 2006, Fred held the top Discovery Grant from the NSERC Mechanical Engineering Committee.

Dr. Habashi is the founder of a public spin-off company, Numerical Technologies International, developing software and providing services for multi-disciplinary applications of Computational Fluid Dynamics. NTI's innovative in-flight icing simulation software is currently used at most major aircraft, helicopter and jet engine companies around the world. He is also co-founder with Captain Gary Wagner, and Chairman, of Scientific Aircraft Accident Analysis

(SA^3), retained by major law firms and aviation insurance companies to use science as a forensic tool in aircraft accident investigation such for the Embraer crash over Monroe, MI, a number of Cessna 208B crashes, and the famous crash of the Concorde in Paris, whose investigation is still ongoing.

Fred takes particular pride in Canadian strengths in CFD and does not hesitate in reminding people that WE are indeed prominent. Computational Dynamics' StarCD was established by David Gosman from UBC; ANSYS' CFX by George Raithby; Analytical Methods Inc.'s VSAERO came from Frank Dvorak from RMC; MAYA technologies codes came from Adam Harris from McGill, and finally, Newmerical Technologies' FENSAP-ICE from McGill.

Finally, Fred is also known for the Habashi CFD Rules, a series of hilarious discipline-self-deprecating quotations and observations that he has presented at conferences and banquets and that are meant to slow us down whenever we take ourselves too seriously.

A Report on CFD 2006

The 14th Annual Conference of the CFDSC was held on the campus of Queen's University in July. By all accounts, it was a huge success! It attracted over 130 delegates from 15 countries.

This conference attracted over 150 abstracts and each of these were sent out to up to three reviewers, and we wish to express our thanks to both the authors and the reviewers for their adherence to strict deadlines, their diligence and significant contributions to ensure the success of this meeting. We introduced for the first time a web-based paper handling procedure, which we believe will help subsequent organizers of this conference; so thanks to the Society for buying the server for this feature.

A conference cannot be mounted without the financial support of sponsors. Our platinum and banquet sponsor is Sun Microsystems. Our gold sponsors, who kindly sponsored lunches, are IBM and Maya Technologies. Bombardier Aerospace sponsored the opening reception. Maya Technologies and Liquid Computing sponsored beverage breaks. Our other valued sponsors are Numerical Technologies, CEI, ANSYS, Upwind Technology, the Faculty of Applied Science, The Queen's Bookstore and the Department of Mechanical and Materials Engineering. We introduced an industrial session, which was well received. The conference also features a job fair and a student paper competition; session participants will be asked to help provide input to this important conference element.

Professors Roger Davis, Ned Djilali, Wagdi (Fred) Habashi, and Gordon Mallinson gave stimulating invited talks. The CFDSC honored Fred Habashi with its Lifetime Achievement Award.

Two students were awarded Best Student Paper: first prize went to Alexander Korobov from the University of Waterloo while second prize went to Christopher Ball from Queen's University. The delegates, using a survey completed during each presentation, ranked these the best.

A short course this year was given by Li-Shi Lou of Old Dominion University and entitled 'Lattice Boltzmann Equation: A Multi-scale Approach for Flow Problems'. The course was highly successful, with 23 participants.

The organizers are currently undertaking the review process of papers selected in part from the recommendations of session chairs at the conference for a special issue of the International Journal of CFD.

Finally, we would like to extend our thanks to the CFDSC for their support and confidence in enabling us to host CFD 2006.

Andrew Pollard (Chair)
Jon Pharoah
Darko Matovic
Conference Organizers
Department of Mechanical and Materials Engineering
Queen's University at Kingston

CFDSC Graduate Scholarship

In 2005, the CFD Society of Canada created a new scholarship for full time graduate students at Canadian universities that are conducting research related to computational fluid dynamics. The field of study may involve the development of CFD techniques and/or the use of CFD techniques to solve a particular problem. The value of the award is $1000 and the CFDSC may award up to three per year.

One of the goals of the Society is to promote research in the field of computational fluid dynamics in Canada and the recognition and support of graduate students and their work is an important part of that objective. The Committee was very impressed by the high quality of research being carried out by graduate students across Canada.

For the first year of this new scholarship there were a number of impressive applications and the Scholarship Committee decided to give the maximum number of awards for the upcoming academic year.

The winners of the CFDSC Graduate Scholarship for the 2006-2007 academic year are:

James McDonald with his work "Application of Gaussian Moment Closure to Micron-Scale Flows" at the University of Toronto.

Catherine Strutt with her work "Analysis of Particle Dispersion Models Using Numerically Simulated Turbulence" at McMaster University.

Amir Baserinia with his work "Grid Adaptation in Low-Speed Laminar Flows" at the University of Waterloo.

Congratulations!

Two-dimensional Flow over Rough Topography: Spectral Analysis

Author Alexander Korobov[1] and Kevin Lamb[1]

Department of Applied Mathematics, University of Waterloo, Waterloo, ON, N2L 3G1, Canada

Email: *akorobov@uwaterloo.ca*

ABSTRACT

The primary goal of this work is to provide a description of the energy cascade in internal waves generated by tidal flow over topography.We have carried out several sets of numerical experiments to model the dynamics of the internal wave field in the deep ocean. A two-dimensional finite volume model solving the non-hydrostatic equations of motion was used. The experiments were performed for different latitudes and various types of topography. Thorough spectral analysis of the obtained data was performed. In particular, the evolution of the two-dimensional spectra for the developing flow was calculated. The spectra of the time-series associated with points fixed in Lagrangian and Eulerian reference frames were also computed. The results were compared to several previous works in the field and appeared to be essentially different from those obtained by MacKinnon and Winters [1] who used a three-dimensional spectral model to investigate the spectral evolution. The distinction between their approach and ours lies in the way we model the bottom forcing. MacKinnon and Winters used a flat bottom ocean and generated internal waves with an upwardly propagating internal tide via a forcing term in the momentum equations, whereas in our simulations we explicitly consider the wave generation process by introducing topography.

1 INTRODUCTION

1.1 Importance

Internal gravity waves generated by tidal flows over rough topography play an important role in deep ocean mixing [5, 6]. As the tidal flow goes back and forth over various topographic features in the deep ocean (ridges, hills, seamounts, etc.), it generates internal waves of the tidal frequency, called internal tides. These waves can propagate in the continuously stratified ocean long distances away from the source. The waves can also nonlinearly interact, so that the mechanical energy, originally concentrated at large scales, cascades down to the small scales and eventually dissipates through turbulence or mixing. The deep ocean mixing transfers some heat from the upper layer of the ocean to the abyssal cold waters, thus sustaining the so-called meridional overturning circulation, in which the water masses from the abyss upwell gradually to the surface of the ocean bringing with it nutrients and chemicals. The described chain of interconnected processes, namely, internal tides, nonlinear energy cascade in the internal wave field, deep-ocean mixing and abyssal circulation, affect the dynamics of climate, marine productivity and chemical dispersal, which are the main applications for the studies of the aforementioned phenomena.

Tide-topography interaction is not the only contributor to the internal wave field. Another important source of energy is the wind. Together, the tides and wind provide the majority of the energy necessary for supporting the abyssal circulation. However, in this study we will be concerned only with tide-topography interaction.

We study the internal wave generation by modeling a two-dimensional tidal flow over idealized rough topography in the deep ocean. Further, we investigate the energy transfer in internal waves by using spatial and temporal spectral analysis. The idea of investigating energy transfer in numerically modeled internal waves is not new [e.g. 1, 2, 3, 4]. However, the previous studies were concerned with the uniform domains representing the interior of the ocean, whereas in our work we introduce topography in the computational domain, allowing explicit wave generation and, thus, more realistic wave field calculations.

1.2 Framework

There are several physical assumptions that we make in this work:

- We assume that the flow is two-dimensional, that is, the flow depends only on one horizontal and one vertical coordinate. However, we calculate all three components of the velocity vector. It is the Coriolis force that gives rise to flow in the third direction.
- We use the f-plane model, where the vertical component of the Coriolis force is assumed negligible and the horizontal component is assumed constant throughout the region of study.
- The fluid is considered to be inviscid, incompressible and linearly stratified.
- The Boussinesq approximation is used, which in the case of incompressible fluid means that the temperature variations in the flow are assumed small.
- We also use the rigid lid approximation in which the motion of the fluid surface is assumed negligible.

Under the assumptions presented above, the equations governing the internal wave dynamics are the two-dimensional incompressible Euler equations under the Boussinesq approximation [7]:

$$\begin{cases} \vec{u}_t + \vec{u}\cdot\vec{\nabla}u + f\vec{k}\times\vec{u} = -\vec{\nabla}p - \rho g\vec{k}, \\ \rho_t + \vec{u}\cdot\vec{\nabla}\rho = 0, \\ \vec{\nabla}\cdot\vec{u} = 0. \end{cases} \tag{1}$$

The unknowns depending on time t and two spatial coordinates x and z are:

- The velocity vector $\vec{u} = (u, v, w)$;
- The normalized density ρ, such that the physical density ρ_{ph} is given by $\rho_{\text{ph}} = \rho_0(1+\rho)$, where ρ_0 is the reference density;
- The normalized pressure p related to the physical pressure p_{ph} by the formula $p_{\text{ph}} = p_a + \rho_0(p - gz)$, where p_a is the atmospheric pressure.

The parameters in the equations are:

- The Coriolis frequency f related to the latitude Θ by the formula $f = 2\Omega\sin(\Theta)$, where $\Omega = 0.73\times 10^{-4}$ 1/sec is the angular velocity of the Earth.
- The gravitational acceleration $g = 9.81$ m/sec^2.

The term $f\vec{k}\times\vec{u}$ represents the Coriolis force with $\vec{k} = (0,0,1)$.

The equations (1) are solved on a fixed domain. The flow is forced with a periodic tidal current $U(t) = U_0\cos(\omega_0 t)$ imposed at the left boundary. The following typical values for the amplitude U_0 and frequency ω_0 were used: $U_0 = 0.025$ m/sec and $\omega_0 = 1.4075\times 10^{-4}$ 1/sec. Free-slip boundary conditions were imposed at the top and bottom of the domain. An outflow boundary condition is applied on the right boundary.

The model is initialized from rest with the linearly stratified density corresponding to the constant buoyancy frequency

$$N \equiv \left(-\frac{g}{\rho_0}\frac{d\rho_{\text{ph}}}{dz}\right)^{\frac{1}{2}} = 10^{-3} \text{ 1/sec}.$$

2 METHODS

2.1 Nonhydrostatic Model

The numerical scheme used to solve equations (1) is the Finite Volume Method based on the second-order projection technique developed by J.B. Bell, P. Colella and J.M. Glaz [8] and extended to stratified flows by J.B. Bell and D.L. Marcus [9] and to quadrilateral grids by J.B. Bell, J.M. Solomon and W.G. Szymczak [10].

2.2 Spatial discretization

The model makes use of the so-called sigma or terrain-following grid. Two staggered grids are employed for the evaluation of scalar and vector unknowns. The two-dimensional domain where the equations (1) are solved consists of the central region with high resolution and the side regions where gradually decreasing horizontal resolution is used, so that fast large-scale waves generated at the centre could propagate away without reflection. The size of the domain is chosen so that waves could not reach the left or right boundary.

The central region is given by

$$R = \{(x,z) \parallel -L/2 \le x \le L/2, -H + h(x) \le z \le 0\},$$

where $L = 8192$ km is the length of the domain, $H = 5$ km is the depth of the ocean, and $h(x)$ defines the topography of the ocean bottom. Typical cell size for the central domain is 100 m in the horizontal by 20 m in the vertical. In this work, the topography was

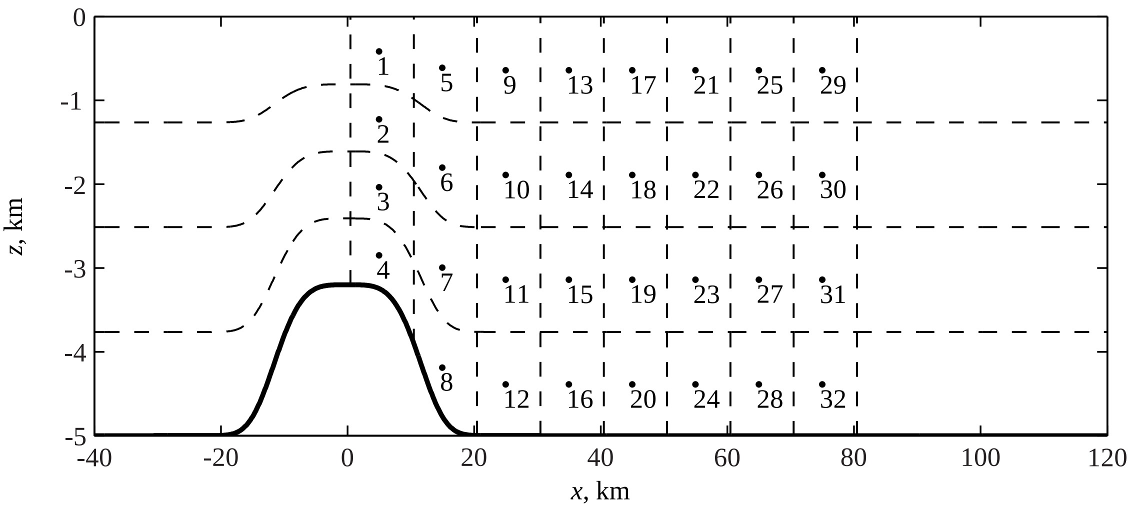

Figure 1: Part of the central region: mooring and starting particle positions.

represented by one or two hills in the central region of the form

$$h(x) = A_0 \exp\left[-\left(\frac{x}{d}\right)^4\right], \qquad (2)$$

where the amplitude A_0 is 1.8 km and the width of the hill $d = 12.5$ km. In the case of one hill, it was placed at the position $x = 0$; in the case of two hills, they were placed at the positions $x = -L/4$ and $x = L/4$, thus, separated by the distance of about 400 km. A part of the central region for one of the runs is shown in Fig. 1.

2.3 Time-stepping

The time-advancing procedure allows variable time step. However, for most runs the maximum time step was set $dt = 30$ s, so that the time step did not actually change throughout the computation. The model was run to compute 30 days of the internal wave dynamics.

2.4 Data

The data produced by each numerical experiment can be classified for convenience into two categories: two-dimensional and one-dimensional. The two-dimensional and one-dimensional spectral analysis techniques applied for each corresponding data type are described in the next two sections. The two-dimensional data are the horizontal velocity fields $u(x,z,t)$ sampled every 12 hours for the time period of 30 days. The one-dimensional data are

- Time-series of the horizontal velocity at the mooring positions sampled every 5 minutes for the time period of 30 days;
- Time-series of the horizontal velocity for the particles launched from the mooring positions, also sampled every 5 minutes for the time period of 30 days.

Time-series produced for moorings and particles correspond to the fixed Eulerian and Lagrangian reference frames, respectively. Only the horizontal component of the velocity was considered because it is the most energetic one.

All the data were calculated for different latitudes (0°N, 20°N, 30°N, and 40°N) and the case of one or two hills.

2.5 Spatial Spectral Analysis (2D)

To see how energy transfers among different wavenumbers in the spectral space, we calculated the two-dimensional power spectra for each snapshot of the horizontal velocity field. To calculate the two-dimensional spectrum, we considered two-dimensional data only from a part of the computational domain, where the bottom was flat and data are distributed uniformly in space. In the case of one hill, it was a subregion lying to the right of the hill:

$$R_{\text{sub},1} = \{(x,z) \mid\mid 20 \text{ km } \leq x \leq L/2, -H \leq z \leq 0\}.$$

For the case of two hills, we used a subregion lying between the hills:

$$R_{\text{sub},2} = \{(x,z) \parallel |x| \le L/4 - 20 \text{ km} \ , -H \le z \le 0\}.$$

As the obtained velocity fields represent wave-like motion, they can be well decomposed into sinusoids with different wavenumbers, so the classical Fourier analysis is applicable.

The calculation of the two-dimensional power spectrum can be split into three steps, as described below.

1. Vertical Transform. Apply the Discrete Cosine Transform (DCT) to the data in the vertical, that is, transform each vertical column of the two-dimensional data matrix. Let $\{f_i\}$ where $i = 0,1,\ldots,N_1 - 1$ be the set of N_1 values for one of the vertical columns. The corresponding vertical coordinates are $z_i = (i+1/2)\Delta z$ where $i = 0,1,\ldots,N_1 - 1$ and $\Delta z = H/N_1$. Then, the DCT of the column $\{f_i\}$, $i = 0,1,\ldots,N_1 - 1$ is given by

$$F_n = \beta(n) \sum_{i=0}^{N_1-1} f_i \cos(m_n z_i) \, , \; n = 0,1,\ldots,N_1 - 1,$$

where $\beta(0) = \sqrt{1/N_1}$ and $\beta(n) = \sqrt{2/N_1}$ when $n = 1,2,\ldots,N_1 - 1$. The vertical wavenumbers m_n are given by

$$m_n = \frac{n\pi}{H}, \; n = 0,1,\ldots,N_1 - 1.$$

2. Horizontal Transform. Multiply each row of the resulting matrix by the Hann window and apply the Discrete Fourier Transform (DFT). The windowing is used to reduce leakage in the spectral estimation. The Hann window appeared to be a good choice for our purposes. Again, let $\{f_i\}$ where $i = 0,1,\ldots,N_2 - 1$ be the set of N_2 values for one of the rows. The corresponding horizontal coordinates shifted for the convenience are $x_i = i\Delta x$ where $i = 0,1,\ldots,N_2 - 1$, $\Delta x = X/N_2$ and X is the horizontal length of the domain. Next, we multiply the row $\{f_i\}$ by the Hann window $g_i = 1/2[1+\cos(\pi x_i/X)]$, so that $f_i := g_i f_i$, where $i = 0,1,\ldots,N_2 - 1$. Then, the DFT of the resulting set $\{f_i\}$ with $i = 0,1,\ldots,N_2 - 1$ is given by

$$F_n = \sum_{i=0}^{N_2-1} f_i \exp(-\hat{\imath} k_n x_i) \, , \; n = 0,1,\ldots,N_2 - 1,$$

where $\hat{\imath}$ is the imaginary unit, and the horizontal wavenumbers k_n are defined by

$$k_n = \frac{2n\pi}{X}, \; n = 0,1,\ldots,N_2 - 1.$$

Only the values F_n with indices $n \le \lfloor N_2/2 - 1 \rfloor$ are needed for the power spectrum estimation, where $\lfloor \gamma \rfloor$ denotes the greatest integer less or equal than γ. The other half with indices $n > \lfloor N_2/2 - 1 \rfloor$ is redundant.

3. Normalization. Let us denote the $\lfloor N_2/2 - 1 \rfloor$-by-$N_1$ matrix resulting from step 1 and 2 as $\tilde{\mathbf{E}}$. The absolute squared values of $\tilde{E}_{ij}$ are proportional to the energy density at the corresponding wavenumbers (k_j, m_i), where $i = 0,1,\ldots,N_1 - 1$ and $j = 0,1,\ldots,\lfloor N_2/2 - 1 \rfloor$. For the continuous two-dimensional power spectrum $E(k,m)$ that we estimate, the total power in the original signal should be equal to the double integral of the spectrum $E(k,m)$ over the whole wavenumber space:

$$\frac{1}{XH} \iint_{R_{\text{sub}}} u(x,z)^2 \, dx\, dz = \int_0^\infty \int_0^\infty E(k,m) \, dk\, dm.$$

Thus, the expression for the two-dimensional power spectrum is given by

$$E_{n_1 n_2} = |\tilde{E}_{n_1 n_2}|^2 \left[\frac{\dfrac{1}{XH} \displaystyle\sum_{i=0}^{N_1-1} \sum_{j=0}^{N_2-1} u_{ij}^2 \Delta x \Delta z}{\displaystyle\sum_{\tilde{n}_1=0}^{N_1-1} \sum_{\tilde{n}_2=0}^{\lfloor N_2/2-1 \rfloor} |\tilde{E}_{\tilde{n}_1 \tilde{n}_2}|^2 \Delta k \Delta m} \right],$$

where u_{ij} is the original horizontal velocity matrix, and the wavenumber discretization steps are $\Delta k = 2\pi/X$, $\Delta m = \pi/H$.

2.6 Temporal Spectral Analysis (1D)

To see how energy transfers among different time frequencies, we calculated the spectra of the time-series associated with the particles and moorings. We used the direct spectral estimation with different windows similar to the horizontal transform described in the previous section. We also used the multitaper spectral estimation described in [11]. The time series are frequently sampled and have a fairly large length, so the spectral estimation allows capturing small-scale features of the spectrum. The quality of the estimated spectra was insured by comparing them.

3 RESULTS

3.1 Velocity

Figure 2 shows a snapshot of the horizontal velocity field near the hill at time $t = 30$ days for the case of 0°N latitude and one hill. The velocity field is dominated by the internal wave beams that criss-cross the

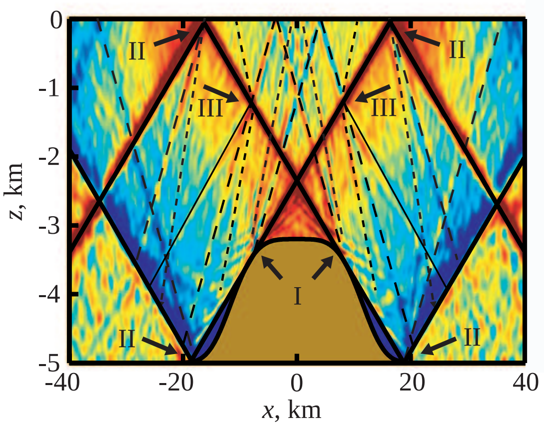

Figure 2: Beam structure and locations of the strongest harmonics generation mechanisms. Solid lines — beams with the tidal frequency ω_0; long dash — beams with the frequency $2\omega_0$; short dash — beams with the frequency $3\omega_0$.

interior of the domain. This beam structure is typical for the fluids with linear stratification. A given beam with a frequency ω has the slope r defined through the linear dispersion relation [12]:

$$r^2 \equiv \left(\frac{k}{m}\right)^2 = \frac{\omega^2 - f^2}{N^2 - \omega^2}, \quad f \le \omega \le N, \tag{3}$$

where (k, m) is the pair of corresponding wavenumbers. The strongest beams seen in the figure are the beams corresponding to the multiples of the tidal frequency: $\omega = n\omega_0$ where $n = 0, 1, \ldots, 7$. However, apart from the tidal harmonics, the velocity field also has less visible beams with subharmonic frequencies different from $n\omega_0$. Their presence is evident at the developing stage of the flow dynamics when a discrete set of beams (both tidal harmonics and subharmonics) with frequencies ranging from the Coriolis frequency f to the buoyancy frequency N emanates from the right and left edge of the hill. As the flow develops, the originally generated beams propagate further away from the hill and interact among themselves producing more beams.

There are the three main mechanisms, responsible for the generation and interaction of different harmonics and subharmonics:

I. Tide-topography interaction near the edges of the hill;

II. Beam interaction near the surface or bottom of the computational domain;

III. Beam interaction in the interior of the domain.

Figure 2 shows the internal wave generation sites corresponding to the mechanism I and some of the strong generation sites corresponding to the type II and III. The flow is highly nonlinear in the vicinity of the strong generation sites and semi-linear away from them.

To get a deeper insight into the energy dynamics, we consider the two- and one-dimensional power spectra of the flow.

3.2 2D Power Spectrum

Figures 3 and 4 demonstrate the two-dimensional power spectra calculated at time $t = 30$ days for the numerical experiments with one and two hills. The dots in the figures show the one hundred most energetic wavenumbers — the majority of energy is concentrated at them.

All of the spectra exhibit a typical ray-like structure, where each ray can be attributed to a certain frequency by the dispersion relation (3). The strongest rays are the ones corresponding to the tidal frequency and twice the tidal frequency, followed by the other tidal harmonics and subharmonics.

The development of the spectra follows the same pattern for all the cases:

1. Originally, energy is generated at the lowest wavenumbers, corresponding to the tidal frequency ω_0;
2. Then, energy gradually cascades to higher wavenumbers along the rays corresponding to different harmonics and subharmonics;
3. Finally, when it has spread in the wavenumber space, it stays semi-steady and does not change the structure very much.

Although the manner in which the spectrum develops is similar for all the cases, the dependence of the spectrum's structure on latitude is rather obscure: for example, one can observe pronounced subtidal frequencies for the cases of 0°N and 20°N latitude, but for the other cases they seem to be absent; the same is for some other subharmonics. The tidal harmonics, however, seem to be present everywhere, so the structural difference is manifested only in the subharmonics.

For the case of 30°N latitude one can also see an energetic patch at low horizontal and high vertical wavenumbers. The energetic patch corresponds to strong beam interaction of the type II happening at

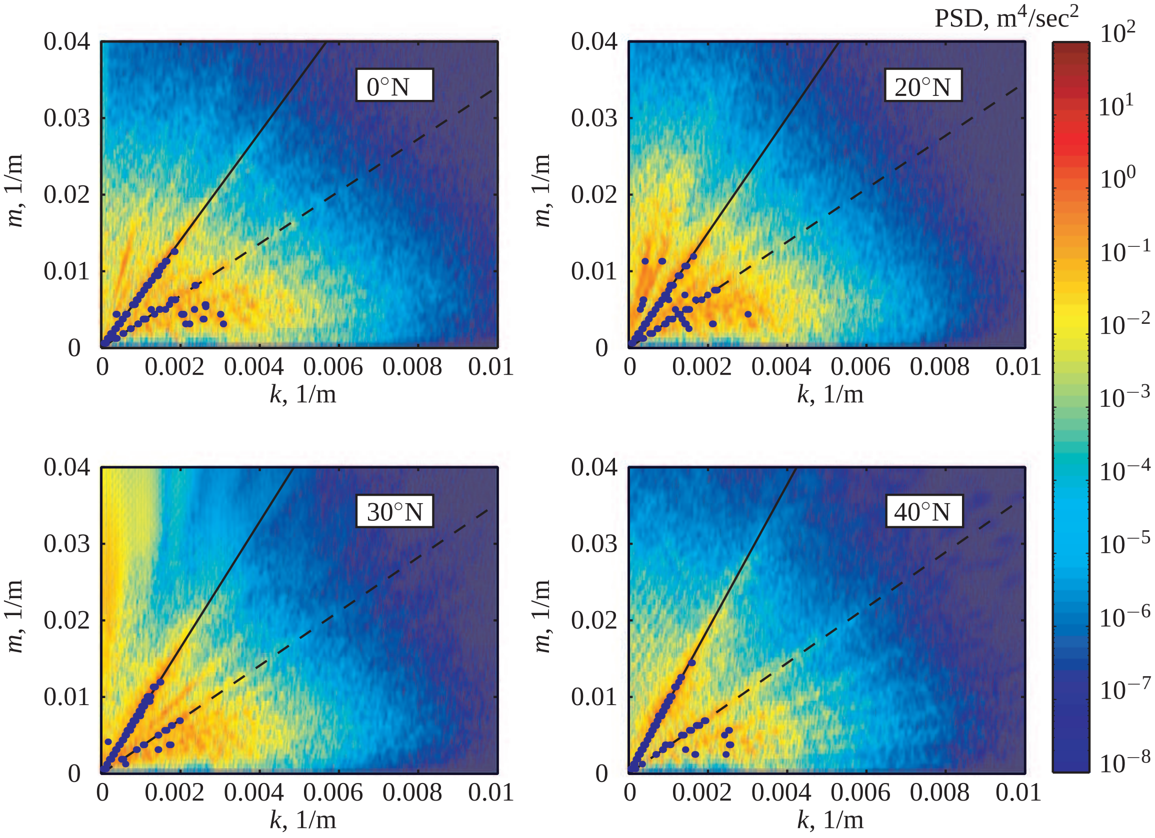

Figure 3: 2D power spectrum at time $t = 30$ days for the case of one hill. Solid line corresponds to the tidal frequency ω_0, dashed line corresponds to the frequency $2\omega_0$. The dots show the most energetic wavenumbers.

the regions where the tidal beams hit the surface or the ground. However, the energy from the patch is not transferred into propagating waves. If we looked at the vertical profile of the horizontal velocity $u(z)$ at those highly nonlinear regions where the beam hits the surface or the ground, we would see that it has a sharp peak-like form. The normal mode decomposition in this case is no longer adequate, and that is why the spectrum has a broad patch.

None of the two-dimensional spectra exhibit separation of scales. This is important as some of the studies on internal waves used the separation of scale assumption to calculate the dissipation rates in the oceanic internal wave fields [e.g. 13].

Comparing the cases of one and two hills, we notice that the values of pronounced frequencies observed in the case of one hill stay the same for the case of two hills. However, the distribution of energy among this energetic set of frequencies is different. According to the most energetic wavenumbers distribution, at 0°N and 20°N latitude energy is evidently distributed more uniformly in the case of two hills. This may suggest that adding another hill enhances the energy flow from tidal harmonics to the subharmonics.

3.3 1D Power Spectrum

Figure 5 demonstrates two average one-dimensional power spectra for particles and moorings estimated with the multitaper method for the case of 0°N latitude and one hill. From the comparison of the one-dimensional spectra, we obtained several results:

- The spectra for particles and moorings are very close to each other, especially at low frequencies. This is important as, in practice, many spectra are obtained for particles, but not moorings, and vice versa, so that it is hard to draw a conclusion concerning the spectrum of the counterpart.
- As expected, we observe strong tidal harmon-

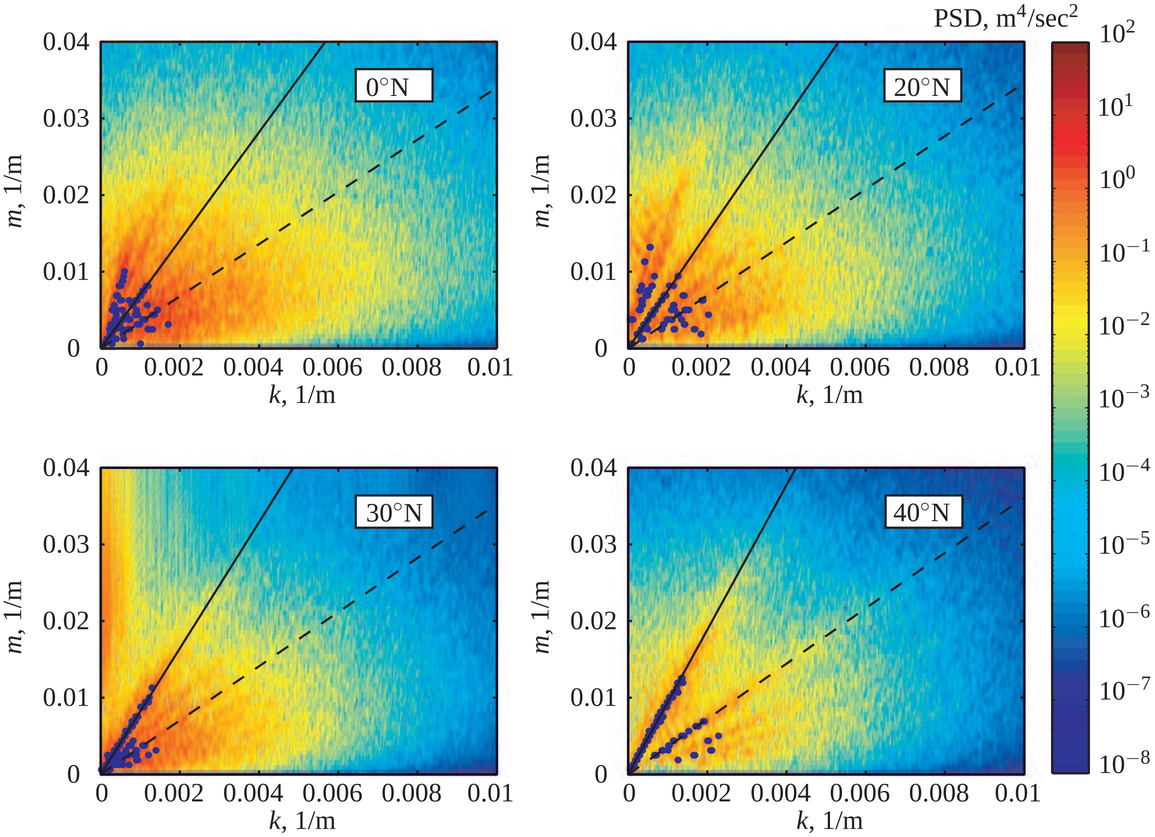

Figure 4: 2D power spectrum at time $t = 30$ days for the case of two hills. Solid line corresponds to the tidal frequency ω_0, dashed line corresponds to the frequency $2\omega_0$. The dots show the most energetic wavenumbers.

ics $n\omega_0$ with $n = 0, 1, \ldots, 7$ which correspond to the frequencies of the strong internal wave beams. However, the energetic tidal frequencies extend even beyond the buoyancy frequency without appreciable change in the spectrum slope. These frequencies most probably correspond to the forced internal waves.

- The spectra have a characteristic exponential slope of -2 (if the units of frequency are taken to be 1/hrs) for the frequencies $f \leq \omega \leq N$. The power law of -2 known from observations and corresponding the internal wave spectrum in the interior of the ocean away from the topography is also shown in the Fig. 5 as a reference.
- There are strong subharmonics present in the spectrum. Their distribution in the spectrum has a self-similar structure, such that each subharmonic forms a triad with any of the tidal harmonics plus another subharmonic. The tidal harmonics themselves also form triads.

We considered the evolution of the one-dimensional power spectrum by estimating the spectrum on the time intervals $t = [0 + \alpha; 20 + \alpha]$ days where $\alpha = 0, 1, \ldots, 10$ days. For all the cases, the energy grows uniformly in all the energetic frequencies, until the spectrum reaches semi-steady state. In this regard, our results are essentially different from the ones obtained by J.A. MacKinnon and K.B. Winters [1]. In particular, for the case of 20°N latitude, there is no transition of the flow from dominantly tidal to lower frequency motion — the tidal frequency always stays dominant (just as is seen from the velocity fields).

4 CONCLUSIONS

We have conducted several numerical experiments modeling internal wave dynamics and used spectral analysis to investigate the associated energy transfers. A number of new features in the internal wave spectra was discovered.

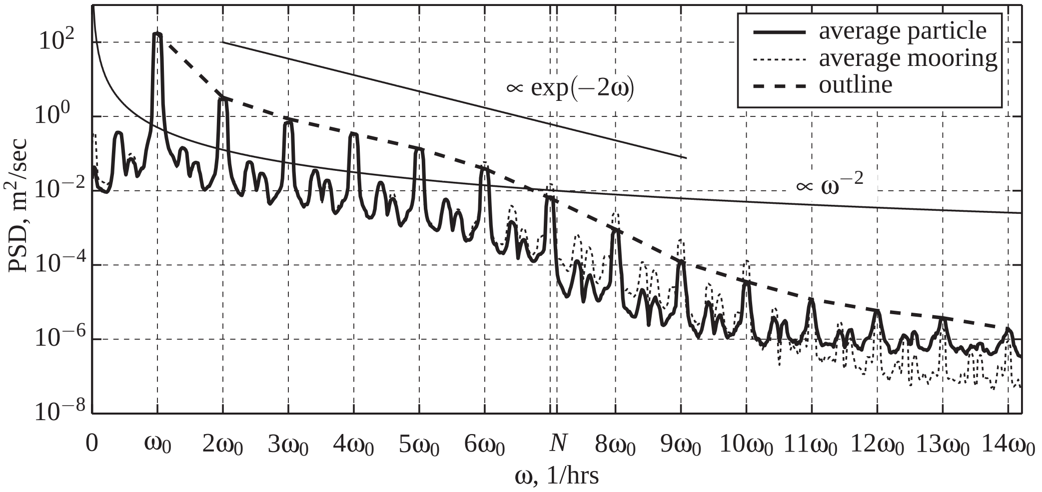

Figure 5: 1D spectrum for the case of 0°N latitude and one hill.

ACKNOWLEDGEMENTS

This work is supported by a Research Network Grant (CLIVAR) funded by the Natural Sciences and Engineering Research Council and the Canadian Foundation for Climate and Atmospheric Science.

REFERENCES

[1] J.A. MacKinnon and K.B. Winters. Spectral evolution of bottom-forced internal waves. *The 13th Aha Huliko at Hawaiian Winter Workshop*, 2003.

[2] T. Hibiya, Y. Niwa and K. Fujiwara Numerical experiments of nonlinear energy transfer within the oceanic internal wave spectrum. *Journal of Geophysical Research*,103(C9):18715–18722, 1998.

[3] R. Furue. Energy transfer within the small-scale oceanic internal wave spectrum. *Journal of Physical Oceanography*, 23:267–282, 2003.

[4] K.B Winters and E.A. D'Asaro. Direct simulation of internal wave energy transfer. *Journal of Physical Oceanography*, 27:1937–1938, 1997.

[5] J.R. Ledwell, E.T. Montgomery, K.L. Polzin, L.C.St. Laurent, R.W. Schmitt and J.M. Toole. Evidence for enhanced mixing over rough topography in the abyssal ocean. *Letters to Nature*, 403:179–182, 2000.

[6] L.St. Laurent and C. Garret. The role of internal tides in mixing the deep ocean. *Journal of Physical Oceanography*, 32:2882–2899, 2002.

[7] K.G. Lamb. Numerical experiments of internal wave generation by strong tidal flow across a finite amplitude bank edge. *Journal of Geophysical Research*, 99:843–864, 1994.

[8] J.B. Bell, P. Colella and H.M. Glaz. A second-order projection method for the incompressible Navier-Stokes equations. *Journal of Computational Physics*, 85:257–283, 1989.

[9] J.B. Bell and D.L. Marcus. A second-order projection method for variable-density flows. *Journal of Computational Physics*, 101:334–348, 1992.

[10] J.B. Bell, J.M. Solomon and W.G. Szymczak. A second-order projection method for the incompressible Navier-Stokes equations on quadrilateral grids. *AIAA 9th Computational Fluids Dynamics Conference*, American Institute of Aeronautics and Astronautics, Buffalo, New York, 1989.

[11] D.B. Percival and A.T. Walden Spectral analysis for physical applications: multitaper and conventional univariate techniques. *Cambridge University Press*, 1993.

[12] K.G. Lamb. Nonlinear interaction among internal wave beams generated by tidal flow over supercritical topography. *Geophysical Research Letters*, 31, L09313, 2004.

[13] F.S. Henyey, J. Wright and S.M. Flatté. Energy and action flow through the internal wave field: an eikonal approach. *Journal of Geophysical Research*, 91:8487–8495, 1986.